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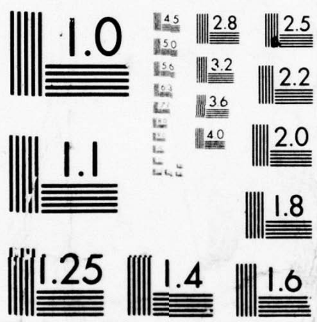
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CRITICAL POINTS AND STREAMLINES IN VISCOUS FLOWS

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In two dimensional flows, stagnation points may appear away from solid boundaries. Numerical, analytical and experimental evidence is provided to support the existence of such "critical" points. The streamline pattern in the neighborhood of critical points is investigated. An intimate relationship between stagnation and separation points is discovered.

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ABSTRACT

In two dimensional flows, stagnation points may appear away from solid boundaries. Numerical, analytical and experimental evidence is provided to support the existence of such "critical" points. The streamline pattern in the neighborhood of critical points is investigated. An intimate relationship between stagnation and separation points is discovered.

1. INTRODUCTION

The present paper is not a report on the solution of a specific problem, nor does it represent a certain amount of effort normally distributed in the last year or two. It is rather a collection of ideas that we have been flirting with for the last eight years, supported by evidence that we were able to collect in this period of time. The topic must be interesting to instructors of fluid mechanics and may appear intriguing to investigators of laminar or turbulent wakes, turbulent boundary layers, etc. It should be made clear to the reader that the present publication does not herald a breakthrough in solving a specific mathematical problem. However, some elementary mathematical justifications are provided and numerical evidence from recent publications of various authors as well as of our group are cited. Some experimental evidence is also supplied, including the results of the initial stage of our present experimental project.

It is felt that some of the ideas presented here may appear quite controversial, and we have struggled to refrain from ungrounded speculation. At some points though, we considered it an insult to our physical intuition, if we were to exclude some "prophetic" comments.

The first two sections that follow are quite straightforward. It is felt that they will not arouse any negative reaction. In these sections we have collected most of the concrete evidence from our numerical results. We also included a straightforward asymptotic analysis which is a modification of the classical slow flow solution about a cylinder, as well as some initial experimental evidence derived via flow visualization. The subsequent sections represent an attempt to generalize the basic ideas involved. We feel that this material is quite innovative. It is definitely a different point of view on a classical problem in mechanics.

We are not sure whether this material will have any effect in the future development of fluid mechanics. In the quite long period that these ideas were kept in rough notes, we did not have time to carry the theory much further nor was any of our other work strongly influenced by them. At the end of the paper the reader will find some ideas and suggestions on what we feel should be done next. We hope that others may find a better way of making use of the present material, in interpreting or solving problems in fluid mechanics.

II. STOKES FLOW ABOUT A ROTATING CYLINDER

One of the most common and appropriate examples for demonstrating that Laplace's equation does not have a unique solution, if inviscid boundary conditions are imposed, is the flow about a circular cylinder. In this case a very simple solution of the linear equation is

$$\psi = U_\infty \left(r - \frac{a^2}{r}\right) \sin\theta \qquad (1)$$

where ψ is the stream function, U_∞ is the free stream velocity, r and θ are polar coordinates and a is the radius of the cylinder. The function $-(\Gamma/2\pi)\ln r$ where Γ is an arbitrary constant may be added to the classical solution without violating the boundary conditions of no-penetration on the wall and no disturbance at infinity. It is well known that this solution corresponds to a vortex of arbitrary strength and the constant Γ, which is the circulation about the cylinder, remains unspecified. The streamline pattern of the potential flow, so well known from classical textbooks of fluid mechanics, is shown in Fig. 1. The only realistic way of generating a rotational field about a circular cylinder is to set the cylinder itself in rotation. The constant Γ then depends upon the angular velocity, ω, of the rotation of the cylinder. The importance of this particular problem is diminished by the fact that the pattern of Fig. 1 is completely violated, if the flow separates from the cylinder. We should therefore assume that the Reynolds number is very small so that no separated bubbles or wakes appear. The solution will be rendered unique, only if consideration is given to viscous effects. The presence of the terms

will allow us to match the potential flow and the moving walls of the cylinder and thus arrive at a unique solution.

The most interesting feature of the complete flow pattern that we can see immediately is that no streamline may attach on the cylinder. The streamline configuration depicted in Fig. 1a is unrealistic to the same extent it is unrealistic to assume that the flow slips on the boundary of the cylinder. There will be no stagnation point on the cylinder since the skin of the cylinder has a transverse velocity $v_\theta = a\omega$ where a is the radius of the cylinder. Instead, there should be a stagnation point away from the wall and within the flow. Arguments based on physical grounds to support the above statement can be found in Section IV. Here we will quickly outline an asymptotic analysis that proves our point. The analysis that follows does not contain many innovative points. In fact it represents a modification of a classical problem, the Stokes flow about a cylinder. The reader will recall that the work of Proudman and Pearson (1957) and Kaplun (1957) on the topic is now considered a classic. For the sake of completeness we will repeat here a few steps of the analysis. Details can be found in the papers cited as well as most of the modern textbooks on Perturbation Theory or Viscous Flows.

Let us assume that the Reynolds number based on the diameter of the cylinder, $R = U_\infty a/\nu$, is a very small number. The inner and outer radial coordinates then are defined as follows

$$r = \frac{\tilde{r}}{a} \quad , \quad \rho = rR \tag{2}$$

where $\tilde{r}$ is the physical distance (see Fig. 2). The dimensionless streamfunction, $\psi(r,\theta)$ in the inner-Stokes layer satisfies the equation

$$\frac{1}{R}\nabla_r^4\psi = \frac{1}{r}\frac{\partial\psi}{\partial\theta}\frac{\partial}{\partial r}(\nabla_r^2\psi) + \frac{1}{r}\frac{\partial\psi}{\partial r}\frac{\partial}{\partial\theta}(\nabla_r^2\psi) \tag{3}$$

where $\nabla_r^2 = \partial^2/\partial r^2 + r^{-1}\ \partial/\partial r + r^{-2}\ \partial/\partial\theta$. If the Reynolds number is absorbed in the streamfunction, then the outer-Oseen layer streamfunction $\Psi(\rho,\theta)$ satisfies the equation

$$\nabla_\rho^4\Psi = \frac{1}{\rho}\frac{\partial\Psi}{\partial\theta}\frac{\partial}{\partial\rho}(\nabla_\rho^2\Psi) + \frac{1}{\rho}\frac{\partial\Psi}{\partial\rho}\frac{\partial}{\partial\theta}(\nabla_\rho^2\Psi) = 0 \tag{4}$$

where $\nabla_\rho^2 = \partial^2/\partial\rho^2 + \rho^{-1}\partial/\partial\rho + \rho^{-2}\partial/\partial\theta$. The appropriate boundary conditions on the wall of the rotating cylinder require that

$$\psi(1,\theta) = 0 \quad \text{and} \quad \partial\psi(1,\theta)/\partial r = u_w \tag{5}$$

where u_w is the dimensionless velocity of the wall. Far away from the cylinder the solution should tend to the undisturbed free stream

$$\Psi(\infty,\theta) \to \rho \sin\theta \tag{6}$$

The Stokes and Oseen solutions should match as follows:

$$\Psi(\rho,\theta) \sim R\psi(r,\theta) \tag{7}$$
$$\rho \to 0 \qquad r \to \infty$$

Solutions to the inner and outer expansions are sought now in the form of asymptotic expansions,

$$\psi(r,\theta;R) = f_0(R)\psi_0(r,\theta) + f_1(R)\psi_1(r,\theta) + \ldots \tag{8}$$

$$\Psi(\rho,\theta;R) = F_0(R)\Psi_0(\rho,\theta) + F_1(R)\Psi_1(\rho,\theta) + \ldots \tag{9}$$

where $f_{i+1}(R)/f_i(R)$ and $F_{i+1}(R)/F_i(R)$ tend to zero as $R \to 0$, for $i = 1,2,\ldots$ The unknown functions ψ_0, Ψ_0, Ψ_1, ... satisfy the following differential equations

$$\nabla_r^4\psi_0 = 0 \tag{10}$$

$$\nabla_\rho^4 \Psi_0 = 0 \tag{11}$$

$$\nabla_\rho^4 \Psi_1 = \frac{1}{\rho}\frac{\partial \Psi_0}{\partial \theta}\frac{\partial}{\partial \rho}(\nabla_\rho^2 \Psi_1) - \frac{1}{\rho}\frac{\partial \Psi_0}{\partial \rho}\frac{\partial}{\partial \theta}(\nabla_\rho^2 \Psi_1) \tag{12}$$

The matching process given by Eq. (7), will provide some of the necessary boundary conditions for ψ_0, ψ_1, ... and Ψ_0, Ψ_1, ... At the cylinder wall and at infinity the appropriate boundary conditions are

$$\psi_0(1,\theta) = \psi_1(1,\theta) = \ldots = 0 \tag{13}$$

$$\frac{\partial \psi_0(1,\theta)}{\partial r} = \frac{u_w}{f_0(R)} \;, \quad \frac{\partial \psi(1,\theta)}{\partial r} = \frac{\partial \psi_2(1,\theta)}{\partial r} = \ldots \to 0 \tag{14}$$

$$\Psi_0(\infty,\theta) \to \rho \sin\theta \;, \quad \Psi_1(\infty,\theta) = \Psi_2(\infty,\theta) = \ldots \to 0 \tag{15}$$

The solution to Eq. (11) that meets the boundary condition (15) is

$$\Psi_0 = \rho \sin\theta \tag{16}$$

and $F_0(R)$ is chosen equal to 1. The solution to Eq. (10) that meets the boundary conditions (13) is

$$\psi_0 = g_1(r)\sin\theta + \sum_{n=2}^{\infty} g_n \sin n\theta + \frac{u_w}{f_0(R)} \ln r \tag{17}$$

where

$$g_1(r) = C_1(r^3 - 2r + \frac{1}{r}) + \frac{1}{2} E_1(2r \ln r - r + \frac{1}{r}) \tag{18}$$

$$g_n(r) = C_n[r^n - nr^{2-n} + (n-1)r^{-n}] + E_n[r^{n+2} - (n+1)r^{2-n} + nr^{-n}] \tag{19}$$

and the C_i's and E_i's for i = 1,2,... are constants. Matching at this stage according to Van-Dyke's (1964) principle requires that 1-term outer expansion (1-term Stokes) ~ 1-term inner expansion (1-term Oseen).

If we express $\psi_0(r,\theta)$ in terms of ρ, expand for small R and retain

the first term of the expansion we get

$$\psi_0(r,\theta) \rightarrow \frac{E_1}{2}\left(-\frac{2\rho}{R}\ln R\right)\sin\theta \tag{20}$$

and matching according to Eq. (7) allows us to choose

$$f_0(R) = \frac{1}{\ln R}\ , \quad E_1 = -1 \tag{21}$$

Notice that the constants C_n and E_n for $n = 2,3,\ldots$ have to vanish since there is no $\sin n\theta$ in the outer solution to match with. Moreover at this stage the quantity $u_w \ln r$ does not appear for $r \rightarrow \infty$ since it is overpowered by quantities like $r \ln r$, r, etc.

Back to the Oseen expansion now, after some algebra we can bring Eq. (12) in the well known form

$$\left(\nabla^2 - \frac{\partial}{\partial\xi}\right)\nabla^2\Psi_1 = 0 \tag{22}$$

where $\xi = xR$ and both x and ξ are coordinates in the stream direction: $x = \tilde{x}/a$, $\xi = \tilde{x}U/\nu$; with $\tilde{x}$ the physical coordinate. Equation (22) is satisfied by

$$\nabla^2\Psi_1 = e^{\xi/2}\sum_{n=1}^{\infty} D_n K_n(\rho/2)\sin n\theta \tag{23}$$

where K_n is the Bessel function of the second kind so that the boundary conditions of Eq. (14) are met. Matching now can be achieved at the level: 1-term inner expansion (2-term Oseen) $\sim$ 2-term outer expansion (1-term Stokes) In the present development we are only interested in the Stokes solution, and we are satisfied to see that matching the ∇^2 of the solutions gives

$$\nabla_r^2\ (\text{1-term Stokes}) = \nabla_r^2(f_0\psi_0) = -\frac{2\sin\theta}{r\ln R} \tag{24}$$

$$\nabla_\rho^2\ (\text{2-term Oseen}) = \nabla_\rho^2(\Psi_0 + F_1\Psi_1) = F_1\nabla_\rho^2\Psi_1 \tag{25}$$

Hence the term $u_w \ln r$ of Eq. (17) is not involved in the matching at this level. To this approximation therefore the Stokes solution reads

$$\psi(r,\theta) = -\frac{1}{2 \ln R}(2r \ln r - r + \frac{1}{r})\sin\theta + u_w \ln r \tag{26}$$

The above analysis holds in the limit $R \to 0$. Let us now define for convenience the small positive quantity $\varepsilon = -1/2 \ln R$. The velocity components then are

$$u_\theta = -\varepsilon(2 \ln r + 1 - \frac{1}{r^2})\sin\theta - u_w/r \tag{27}$$

$$u_r = \varepsilon(2 \ln r - 1 + \frac{1}{r^2})\cos\theta \tag{28}$$

Let us further assume that for small wall velocities, u_w, the disturbance due to rotation is small. In particular we will search for a point within the flow, where both the velocity components u_r, u_θ vanish. Let the polar coordinates of this point, which is a stagnation point, be r_s, θ_s. For small u_w, it is reasonable to assume that the stagnation point should be very close to the wall, that is $r_s = 1 + \ell$ where $\ell << 1$. The component u_r vanishes at $\theta = 90°$ and $270°$. For these values of θ, if one requires that u_θ also vanishes, one can arrive, in the limit $\ell \to 0$, at the approximate expression

$$\ell \simeq \frac{1}{1 \mp 2\varepsilon/u_w} \quad \text{for } \theta = 90° \text{ or } 270°\text{, respectively} \tag{29}$$

If $\theta = 90°$ then for $u_w < O(\varepsilon)$, $= O(\varepsilon)$ or $> O(\varepsilon)$, ℓ would be large negative, negative or positive of order 1, respectively. However, this quantity cannot be negative, nor can it be of order 1 since this would violate our assumption. If $\theta = 270°$ and $u_w = O(\varepsilon^2)$ or smaller, Eq. (29) represents an acceptable solution and $\ell = O(\varepsilon)$. We therefore arrived at the conclusion that if

the angular velocity of the cylinder is small there exists a stagnation point away from the wall, at the point

$$r_s = 1 + \frac{u_w}{u_w - 2\varepsilon} \quad , \quad \theta_s = 270° \tag{30}$$

It is well known that the simplest streamline configuration of a stagnation point for viscous or inviscid flow is that of a saddle point. With the above information we can make a rough sketch of the streamline pattern. Two of the streamlines that emanate from the stagnation point will extend all the way to infinity. One arrives from $-\infty$ and the other leaves our stagnation point and goes to $+\infty$ (see Fig. 3). To complete the saddle point configuration we need two more critical streamlines that emanate from the saddle point. The simplest topography that would not contradict the sense of rotation of the cylinder, is the one shown in Fig. 3. Notice that these two streamlines will have to meet at the top of the cylinder and thus confine a certain amount of fluid in a ring that surrounds the cylinder. This amount of fluid will stay forever with the cylinder and rotate with it. The streamline pattern depicted in Fig. 3 is not new. As far as the topography of the streamlines are concerned, it is identical with the pattern of Fig. 1b. The pattern of Fig. 1b though represents a solution to the Euler equations and holds for a large enough angular velocity that would give to the field a circulation larger than $4\pi U_\infty a$. The pattern of Fig. 3 represents the Stokes flow about a cylinder rotating with infinitesimally small angular velocity.

At this point we can make some interesting observations. For zero angular velocity and for any value of the Reynolds number there are two stagnation points, both on the skin of the cylinder, at $\theta_s = 0$ and 180°. For slow flow and for any angular velocity, no matter how small, there is

only one stagnation point at $r_s = 1 + \ell$ and $\theta_s = 270°$. It is obvious that there is no continuity in the phenomenon. The stagnation point we have discovered at $\theta_s = 270°$ does not tend to $\theta = 0°$ or $180°$ as $\omega \to 0$. One would be tempted not to call this point a stagnation point, at least not in the usual sense, even though at this point the velocity vanishes. Nevertheless our stagnation point is distinct by one more significant feature, characteristic of stagnation points. The streamline that separates the on-coming stream into two parts, the one that flows above and the one that flows below the body, bypasses the skin of the body and stagnates into our point. A similar stagnating streamline emanates from this point and plays the same role in the right half of the flow field (see Fig. 3). We should refer to such streamlines as stagnation or critical streamlines. One would be tempted to call such streamlines separation streamlines. The term would not be totally unjustified as we will see in the sections that follow.

III. SEPARATION OVER MOVING WALLS

The gist of the present material and the cornerblock of the development that follows is not ours. The whole effort was inspired by the Moore-Rott-Sears conjecture on the streamline pattern near separation over a moving near wall. Moore (1958,1959), Rott (1956, 1964) and Sears (1956) have pointed out in the fifties,that the vanishing of the skin friction is not an appropriate criterion for separation over moving walls. Instead they conjectured that the streamline pattern in the neighborhood of separation, configuration as shown in Figs. 4, 5 and 6 for fixed walls, walls moving downstream and upstream respectively. Physical arguments to support such conjectures are simple and quite convincing. If we assume that for large enough Reynolds numbers the motion of the skin does not affect the outer flow, then the separating pattern, consisting of regions I and II (see Fig. 4), should retain their character as shown in Figs. 5 and 6. It should be recalled that the flow in the recirculating bubbles or the wake is essentially stress-free and therefore viscosity is important only in thin wall layers or free shear layers. Within the boundary layer though the flow should remain attached to the skin and therefore a portion of the oncoming flow of Fig. 5 should stay with the wall and the flow bifurcates at separation. Based on the same physical considerations may argue that the flow approaching the point of separation from within the wake should bifurcate as well. A stagnation point within the flow then appears and a set of four stagnation streamlines separate the flow field into four distrinct regions: the regions I, II, III and IV. Such streamlines will be referred to in the sequel as critical streamlines. Similar arguments may be provided in support of the configuration of Fig. 6.

Moore, Rott and Sears suggested that the proper criterion for separation in these cases is

$$\frac{\partial u}{\partial y} = 0 \qquad \text{at} \qquad u = 0 \tag{31}$$

where u is the velocity component parallel to the wall and y is the coordinate perpendicular to the wall. This criterion,often referred to in literature as the MRS criterion, is tacitly based on the assumption that the critical streamlines at the stagnation point are perpendicular and parallel to the wall respectively. Such an assumption appears reasonable for the case of downstream moving walls, if the Reynolds number, R, is very large, since then all viscous effects, including the streamline patterns of Figs. 5 and 6, will be confined to a very thin layer next to the wall. The thickness of the viscous layer, including the neighborhood of separation, tends to zero as the Reynolds number tends to infinity, with some power of R (Stewartson, 1972). With three of the critical streamlines directed downstream, it is reasonable to assume that the critical streamlines are parallel and perpendicular to the wall respectively, at least for the case of a downstream moving wall, as depicted schematically in Fig. 7. These arguments do not seem to be plausible for the case of an upstream moving wall and indeed its validity has been recently questioned (Fansler and Danberg, 1975, 1976; Tsahalis, 1976; Williams, 1977).

The first experimental evidence in support of the above statements were provided first by Ludwig (1964) and his colleagues (Brady et. al. 1965; Brady et. al., 1967). Experiments were performed in a wind tunnel and velocity profiles about a rotating cylinder were measured using hot wire anemometers. The results provided substantial evidence that the criterion given in Eq. (31) is correct, at least for the case of a downstream moving wall. Ludwig et. al. have constructed streamline patterns, on the measured velocity fields and

verified the flow patterns of Figs. 5 and 6 up to the point of separation. However their Reynolds numbers were high and the wake region fully turbulent. As a result the saddle point configuration was not completely recovered from the side of the wake.

Numerical evidence for the existence of a stagnation point within the flow was furnished originally by Thoman and Szewczyk (1966,1969) who integrated numerically the full Navier Stokes equations. Their work seems to indicate the existence of one saddle point for very low Reynolds number which shifts slowly towards $\theta = 0$ as the speed of rotation increases.

A few years later Telionis & Werle (1973) and Tsahalis and Telionis (1973) integrated numerically the boundary layer equations. The reader is cautioned here to the fact that the boundary layer equations are not an appropriate model of the flow in the neighborhood of separation, since the boundary layer hypothesis is violated a few boundary-layer thicknesses upstream of separation. It is well known though, that if one continues the integration of the boundary layer equations, disregarding the fact that they do not represent the actual flow, one eventually encounters a singularity at the station of zero skin friction. The mathematical features of the separation singularity have been extensively studied in the past few decades (Brown & Stewartson, 1969). It is a well known experimental fact that at separation the streamlines turn sharply into the flow and the wall shear vanishes. The boundary layer equations therefore simulate quite accurately the phenomenon for steady flows over fixed walls. Telionis and Werle (1973) have shown numerically that for the case of downstream moving walls, integration can be carried beyond the point of vanishing wall shear and up to a station where the u-component of the velocity and the velocity gradient vanish away from the wall, that is the criterion of Eq. (31)

is met. At the same station a separation singularity appears. The regions I and III of the flow pattern of Fig. 5 were therefore numerically reproduced. The boundary layer equations proved again to be a valuable model, although there is a slight discrepancy, because the normal component of the velocity remained positive across the boundary layer and therefore the point $u = \partial u/\partial y = 0$ is not truly a stagnation point. The only encouraging feature of the boundary-layer solution is that the normal component remains quite small below the point $u = 0$ but grows violently above it. This supports the earlier claim (Sears & Telionis 1971) that in this case the singularity is centered away from the wall. Moreover this numerical investigation has indicated that the Goldstein singularity is not a feature that always accompanies the point of zero skin friction. The numerical integration passes through the point $\partial u/\partial y = 0$ at $y = 0$ with no evidence of singular behavior. The familiar properties of the singular behavior appear at the MRS station. The fact that the point of zero *skin friction is not a proper criterion for separation* has left us unable to predict the location of separation by integrating the laminar or turbulent boundary layer equations. The above evidence though indicates that the boundary layer equations have the capability to signal the location of separation which also marks the extent of their validity.

Most recently Fansler and Danberg (1975) reconsidered the problem and indicated that the MRS criterion is incompatible with the boundary layer equations for an upstream moving wall. Their arguments do not preclude the MRS pattern for the actual flow. Tsahalis (1976) has also performed calculations using the time dependent boundary-layer equations in order to verify that the criterion of Eq. (31) is approximately valid for flows over upstream moving walls. More detailed arguments for flows over moving walls the reader

will find in Sears and Telionis (1971,1975).

We recently undertook to design an experimental system in order to investigate qualitatively the neighborhood of unsteady separation. Our flow visualization method is based on surface observation of particles. Among the first few test cases, we have investigated the steady flow over a rotating cylinder for Reynolds numbers of the order of 100 to 300. Fig. 8 shows the two recirculating bubbles for a fixed cylinder. In Fig. 9 we have blown up a detail of the flow field in the neighborhood of separation over a downstream moving wall. The four regions I, II, III and IV as defined in Fig. 5 are clearly distinguishable. In Fig. 10 we have captured the stagnation region and the neighborhood of separation over the upstream moving wall. This is the first clear evidence of the existence of saddle points at separation. More information on the experimental results the reader will find in the sister paper of the present report (Telionis & Koromilas 1977).

IV CRITICAL POINTS AND CRITICAL STREAMLINES

The family of curves

$$\frac{dx}{dt} = a_1 x + b_1 y \tag{32}$$

$$\frac{dy}{dt} = a_2 x + b_2 y \tag{33}$$

has a critical point at the origin, which is a saddle, a center or a nodal point if the quantity

$$J = a_1 b_2 - a_2 b_1 \tag{34}$$

is negative zero or positive respectively (Kaplan, 1958). Saddle point configurations of streamlines have been studied before, both for viscous and inviscid flows. Solutions of the Euler equations that describe the flow in the neighborhood of a stagnation point for example are classical. The viscous recirculating bubbles shown in Fig. 8, for low Reynolds numbers and the stagnation point C have been well known for long and investigated experimentally (Schlichting, 1968). Kovasznay (1952) has studied a periodic viscous flow where saddle points of the kind of point C (Fig. 8) where captured. Proudman and Johnson (1962) considered the impulsive start of a blunt cylinder and showed how a saddle point is generated in the neighborhood of the rear stagnation point. The Moore-Rott-Sears point of separation is also a saddle point recently captured numerically and experimentally as described in the previous section. To our knowledge no nodal-point configurations of streamlines are possible unless one would consider the degenerate case of a dipole or multipole.

Based on the information we have gathered up to now we can proceed with a sequence of simple statements and conclusions. In the material that

follows in the present section,the reader will not find any rigorous proofs. He will rather find a new interpretation of some more or less known ideas and a few propositions that are based on common sense and the evidence that we collected in the earlier sections.

Let us first synthesize the fragmented information we have about the rotating cylinder. Once again assuming that the frequency of rotation does not eliminate the two recirculating bubbles altogether, we arrived at the conclusion that the two points of separation become saddle points, of the kind depicted schematically in Figs. 5 and 6 and experimentally captured in Figs. 9 and 10. Let us now turn our attention to the "stagnation streamline". As such we will define the streamline that separates the oncoming flow into two flow regimes: The one that eventually flows above the cylinder and the one the flows underneath the cylinder. For no rotation this streamline attaches at the cylinder at $\theta = 0$. If the skin of the cylinder is in motion, this is obviously not possible. There will be no stagnation on the skin of the cylinder. If the stagnation streamline were to pass beneath the bottom saddle point, point E, then we are forced to admit that a portion of the fluid above this line turns around and eventually passes above the cylinder as shown in Fig. 11. This would imply the existence of a fourth saddle point, the point F. The critical streamlines that emanate from E then are not compatible with the streamlines from F, because the fluid between the streamline a and the streamline b should be directed towards E. This could not be possible, unless there existed yet another saddle point between a and b. One will soon find out that a compatible configuration would require an infinite number of saddle points. As always in such frustrating cases, we are tempted to over-simplify things by introducing an axiom. Let it be here the "axiom of least

number of saddle points". In the spirit of this axiom we can argue that the stagnation streamline dives into the critical point E. In a similar way we conclude that the critical streamline b dives into the saddle point D (see Figs. 8 and 11) and before long one arrives at the pattern depicted in Fig. 12. Flow visualization of this pattern is shown in Fig. 13. The above arguments by no means provide a proof of the pattern of Fig. 12. The reader will find it interesting though to try to construct different streamline patterns, compatible with the saddle point configurations of separation, which I consider well established. He will soon find that the pattern proposed in Fig. 12 is the only acceptable to the intuition of a fluid mechanicist. Experimental evidence in support of the pattern of Fig. 12 was provided only recently, as described in the accompanying paper. A certain amount of fluid, as shown in Fig. 12, is trapped with the cylinder and stays forever with it. This is true for a rotating cylinder even if there are *no* separated bubbles (see Fig. 3). One may argue in a similar way for the pattern of this Figure. Ludwig (1964) has reported indeed that in his smoke visualization experiments, he observed a thin layer of smoke that persisted around the cylinder.

Some straightforward conclusions now can be drawn. No matter how small the frequency of rotation is, the classical stagnation points "jump" from $\theta = 0$ and 180^0 (see Fig. 2 for notation) to the separation points, that is the points D and E (Fig. 12). In fact we should probably consider the classical configuration of Fig. 8 as a degenerate case that one can achieve only for the very special case of $\omega = 0$ and not even for $\omega \to 0$. How far the point of stagnation will jump and which points will be displaced more, so that points A and E, and B and D of Fig. 8 will coalesce respectively is an open question.

An attempt to answer these questions is found in the accompanying paper. Evidence that such a phenomenon occurs is given in Fig. 13. In this figure the narrow region of reversing flow that has been formed between the point $\theta = 180^{o}$ and the point of separation is clearly shown. Considering the relationship between steady separation over moving walls and unsteady separation over fixed walls (Sears & Telionis 1971,1975, Williams 1977), the reader should appreciate the importance of this statement, which is further reinforced by the fact that in real life one encounters complex situations, that often involve moving boundaries and strong unsteady effects.

The reader should further note that with a clockwise rotation, the forward stagnation point moves to the bottom saddle point, that is, in a direction opposite to the motion of the skin. This is true for both cases of Figs. 3 and 12. In general therefore a streamline pattern will contain a certain number of saddle points. If there is no separation, there will be one saddle point as shown in Fig. 3. If there is separation there will be at least three critical points as shown in Fig. 12. One final comment: For the general case we are considering, that is for ω not identically equal to zero, a point of stagnation and a point of separation cannot be distinguished. They both appear as saddle points and one saddle point may often play the role of both. That is, it receives the stagnation line and gives rise to a separation line.

The next question we can pose to ourselves is what happens for the next larger range of Reynolds numbers. It is well known that for $250 < R < 1000$ the wake of a cylinder, or in general a bluff body forms a von Kármán vortex street which in an inertial frame appears to consist of an infinite array of vortices as shown in Fig. 14. In our terminology the pattern of Fig. 14

consists of an infinite array of critical points of the "center" type. Let us now attempt to construct the streamline pattern that an observer riding the cylinder would see. Let us assume that for the viscous vortices generated by the cylinder, the velocity profile at a vertical cross section attains values larger than the speed of the cylinder, U_∞. That is, in Fig. 15a, let us assume that the speeds of the points A and B are equal to the speed of the cylinder, $u_A = u_B = U_\infty$. Superposing the constant velocity vector that will allow us to ride with the cylinder, the points A and B will become critical points and in particular point B will be a saddle point and point A will be a center as shown in Fig. 15b. It is an elementary exercise to assume a velocity field, compatible with the streamline pattern of Fig. 14 that satisfies the above assumption and transform it by a constant vector $\overline{U}_\infty$. We then arrive at a pattern similar to that of Fig. 15b, which can be used to synthesize the pattern of Fig. 16. Thoman and Szewczyk (1966) and Fromm and Harlow (1963), solving numerically the full Navier Stokes equations and later Davies (1975) using a conditional averaging technique, generated flow patterns identical to those of Fig. 16, although the saddle points were not identified as such (see Fig. 17). It is very interesting to note that in the flow pattern of Fig. 16 we can identify three distinct areas. Area A is the outer flow that is directed downstream and tends to the uniform undisturbed flow as we move away from the wake. Area C is made up of vortices trapped in closed loops that are stationary and do not participate in the downstream motion. Disregarding the effect of dissipation momentarily, we discover that placing a blunt body in a free stream generates an infinite wake, littered with vortices that are fixed with respect to the inertial frame. Most interesting also is the area B which represents a portion of the flow that travels consecutively above and below the vortices C.

In fact flow A never comes in contact with flow C and flow B is the only agent available to transfer vorticity to the vortices C.

The von Kármán vortex street is a typical example of a steady-state wake field. The process in which this wake is generated is clearly unsteady. Vortices grow in the lee-side of the cylinder and are shed downstream in an alternating fashion. The next question we would like to pose to ourselves then is the following. What would be the streamline pattern for a wake bubble that grows with time? Consider the flow pattern of 18a and assume that the dead fluid in the bubble is dyed blue, whereas the free and alive stream supplies us with red fluid. In order for one of the bubbles to grow, clearly there must be some exchange of mass. Red fluid must enter into the bubble, "die" and become blue. The mass exchange could occur as shown in Fig. 18b. The situation is similar to the one of a free vortex (see Figs. 19a and b). In both cases the character of the saddle point of separation remains unchanged but the critical point at the center of the bubble change from a "center" to a "focus". Notice also that at each instant there exist two streamlines that originate at $-\infty$ and stagnate at the saddle point. One reaches the stagnation point directly and the other only after surrounding completely the recirculating bubble. The above discussion is definitely reminiscent of turbulent flows, turbulent boundary layers etc. After all turbulence is made up of a large number of eddies that are convected with the mean flow, continually changing their size and vorticity. The patterns of Figs. 18 and 19 could be considered as a simplified picture of turbulent flow.

To complete our account of separation over walls with nonvanishing boundary conditions, let us consider the case of blowing and suction. A

numerical investigation of the problem, within the framework of the boundary layer equations, has proved that a Goldstein singularity is again present at the M.R.S. point (Tsahalis & Telionis, 1973). Consider first the case of suction. Fluid will be sucked across the surface from the free flow and the wake. There exists therefore a streamline that separates the two regions as shown in Fig. 20. Another critical streamline separates the free stream into two regions: the region I that eventually flows above the wake and the region II that is sucked by the wall. Arguing in a similar way we conclude that the streamline pattern in the neighborhood of separation should be that of a saddle point as shown in Fig. 20. For the case of very low Reynolds number and assuming that the flow is steady the flow field about a cylinder with uniform suction should be as shown in Fig. 21. The fluid that is sucked by the porous walls is red fluid, since the size of the bubbles and therefore the amount of the blue fluid is conserved. Again two critical streamlines coming from $-\infty$, arrive at critical points thus defining three distinct regions: the region II of the fluid that is sucked in before separation, the region I that coincides with region IV and corresponds to the fluid that will be sucked in after separation and the region V of the red fluid that is free to move about the cylinder and its wake and flow further downstream. Notice that the slightest suction is enough to remove the bubble with the contaminated blue fluid from its contact with the skin of the cylinder. The streamline pattern for the case of blowing and always for very low Reynolds numbers is shown in Fig. 22. Notice further that the separated bubbles are fully submerged in the flow that emanates from the cylinder.

Identification of a critical point numerically would require identification of zeros in the velocity field and calculation of the Jacobian given in

Eq. (34). If the origin of the coordinates is the critical point under consideration, then in the neighborhood of the origin we can expand as follows

$$\frac{dx}{dt} = u = x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + \ldots \tag{35}$$

$$\frac{dy}{dt} = v = x\frac{\partial v}{\partial x} + y\frac{\partial v}{\partial y} + \ldots \tag{36}$$

and the Jacobian is approximately given by the formula

$$J = \frac{\partial u}{\partial x}\frac{\partial v}{\partial y} - \frac{\partial u}{\partial y}\frac{\partial v}{\partial x} \tag{37}$$

Consider now the Navier Stokes equations for incompressible two dimensional flow

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\nabla^2 u \tag{38}$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \nu\nabla^2 v \tag{39}$$

Taking the x and y derivative of Eqs. (38) and (39) respectively, using the continuity equation and evaluating at the origin we arrive at

$$\frac{\partial^2 u}{\partial x\partial t} - \frac{\partial u}{\partial x}\frac{\partial v}{\partial y} + \frac{\partial v}{\partial x}\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial^2 p}{\partial x^2} + \frac{\partial}{\partial x}\nabla^2 u \tag{40}$$

$$\frac{\partial^2 v}{\partial y\partial t} + \frac{\partial u}{\partial y}\frac{\partial v}{\partial x} - \frac{\partial u}{\partial x}\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial^2 p}{\partial y^2} + \frac{\partial}{\partial y}\nabla^2 v \tag{41}$$

The Jacobian can now be readily calculated

$$J = \frac{1}{\rho}\nabla^2 p - \nu\left(\frac{\partial}{\partial x}\nabla^2 u + \frac{\partial}{\partial y}\nabla^2 v\right) \tag{42}$$

Batchelor (1956 a,b) has investigated the uniqueness of solutions to the Euler equations viewed as a limit of the Navier Stokes equations for $\nu \to 0$.

Batchelor's work permits us to make the following statement: Any streamline pattern derived by solving the Navier Stokes equations is topographically an acceptable pattern for a solution of the Euler equations. Or equivalently: the topography of critical lines and critical points is the same for the solution of the Navier Stokes equations and its sister solution of the Euler equations. Assuming then that $\nu \to 0$, the Jacobian at a critical point of an Euler field becomes

$$J = \frac{1}{\rho} \nabla^2 p. \qquad (43)$$

For a stagnation point $u \propto y$ and $v \propto x$; hence J is negative and the critical point is a saddle point, as is well known.

V. CONCLUSIONS AND RECOMMENDATIONS

Recapitulating we put here together a few simple conclusions expressed in an unusual terminology that will emphasize the spirit of the present investigation.

a. Two or more streamlines may cross each other at a point, provided at this point they reverse their direction. Such streamlines we call critical streamlines and their cross sections critical points.

b. For steady flow about a rigid body in a simply connected region with zero suction, one critical streamline goes to $-\infty$ and one to $+\infty$. All other critical streamlines start and terminate on critical points, on or off the walls of the body.

c. The critical points could be saddle points or centers for steady flow. For unsteady flow a focus is also possible.

It is felt that these ideas may contribute in understanding the phenomena of turbulent flows, wakes, duct flows and boundary layers. It is well known that the response of relatively large eddies is almost inviscid. The present work may lead to simple models that would simulate correctly the mechanisms of mass exchange between eddies of different sizes and their dynamic interaction. It is believed that the turbulent field would then be represented by a system of isolated free vortices that would interact with each other while simultaneously being convected downstream.

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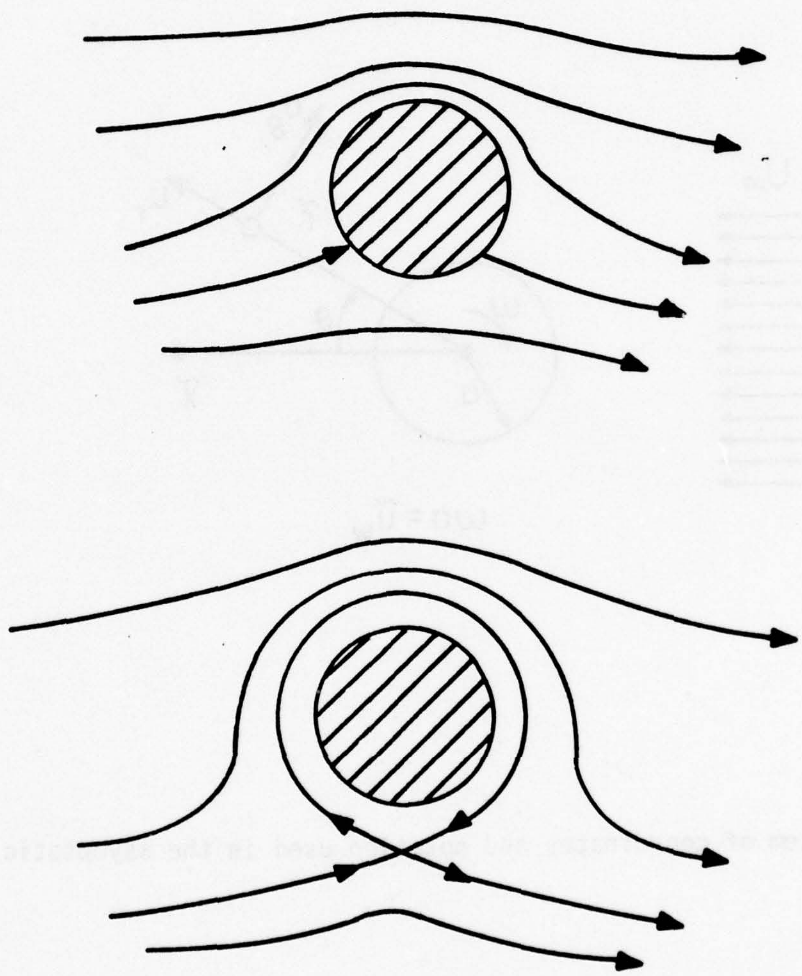

Fig. 1 Schematic sketch of potential flow about a cylinder. a. with circulation $\Gamma < 4\pi U_\infty a$; b. with circulation $\Gamma > 4\pi U_\infty a$.

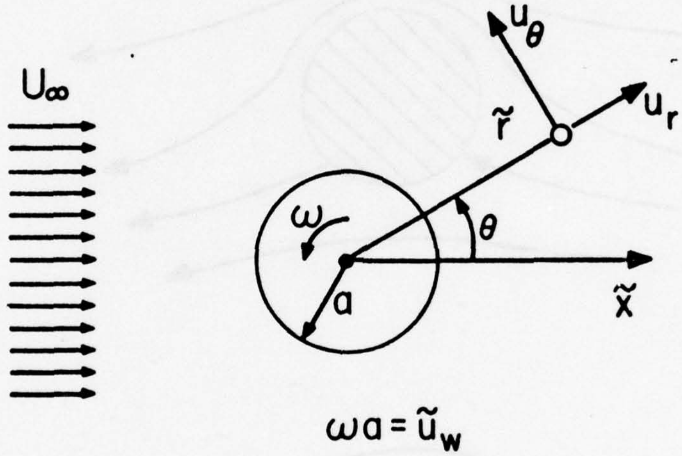

Fig. 2 System of coordinates and notation used in the asymptotic expansion.

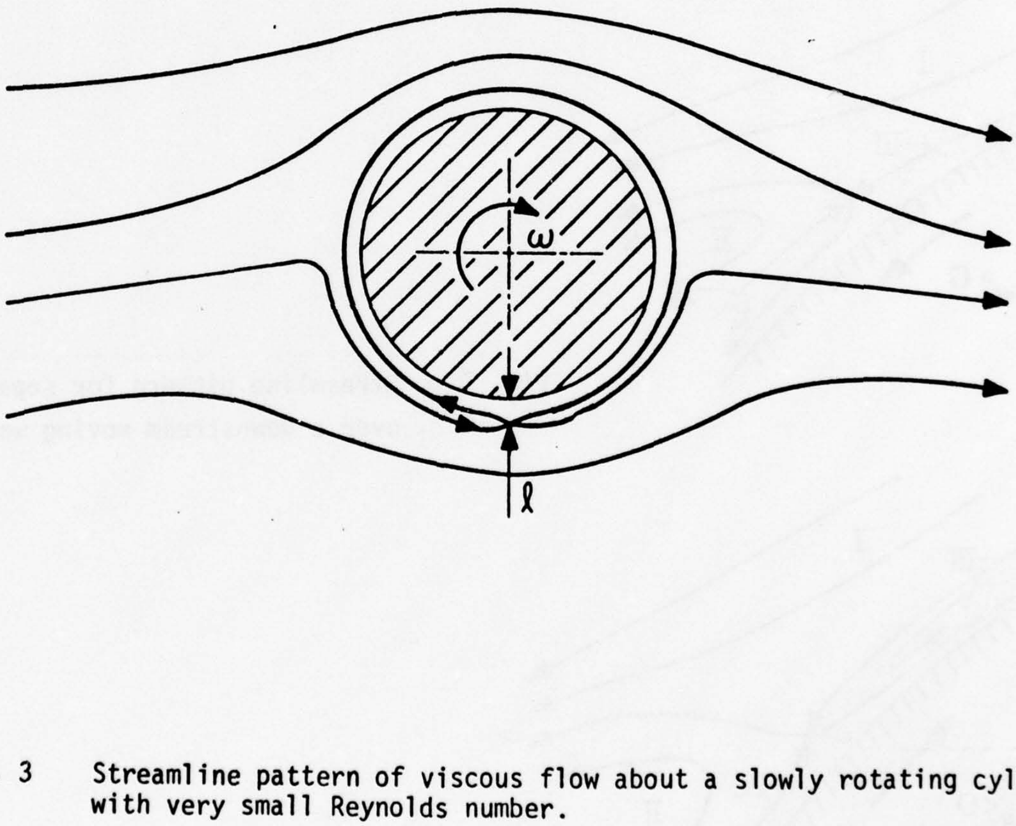

Fig. 3   Streamline pattern of viscous flow about a slowly rotating cylinder, with very small Reynolds number.

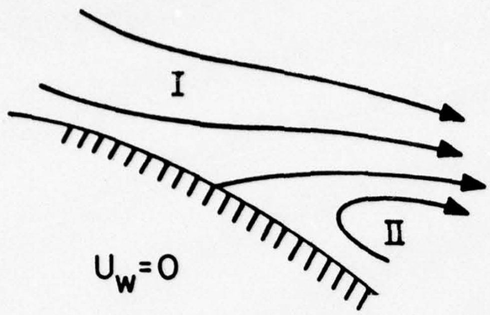

Fig. 4 Streamline pattern for separation over a fixed wall.

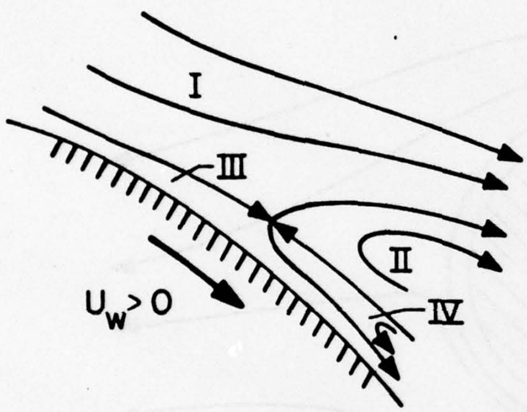

Fig. 5 Streamline pattern for separation over a downstream moving wall.

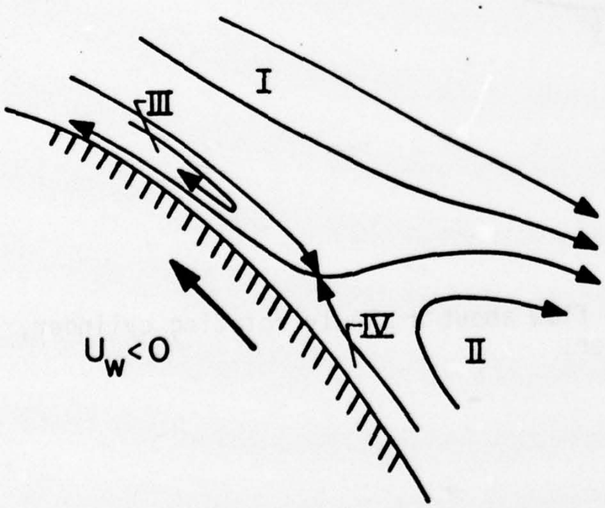

Fig. 6 Streamline pattern for separation over an upstream moving wall.

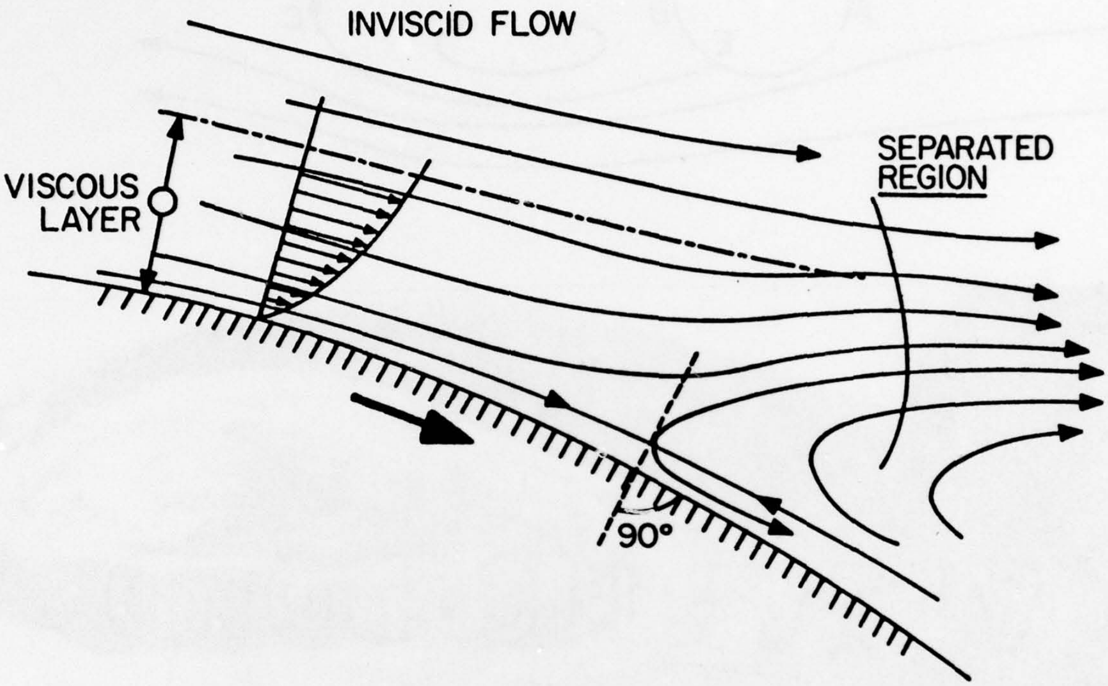

Fig. 7 Streamline pattern for separation over a wall moving downstream with infinitecimal velocity.

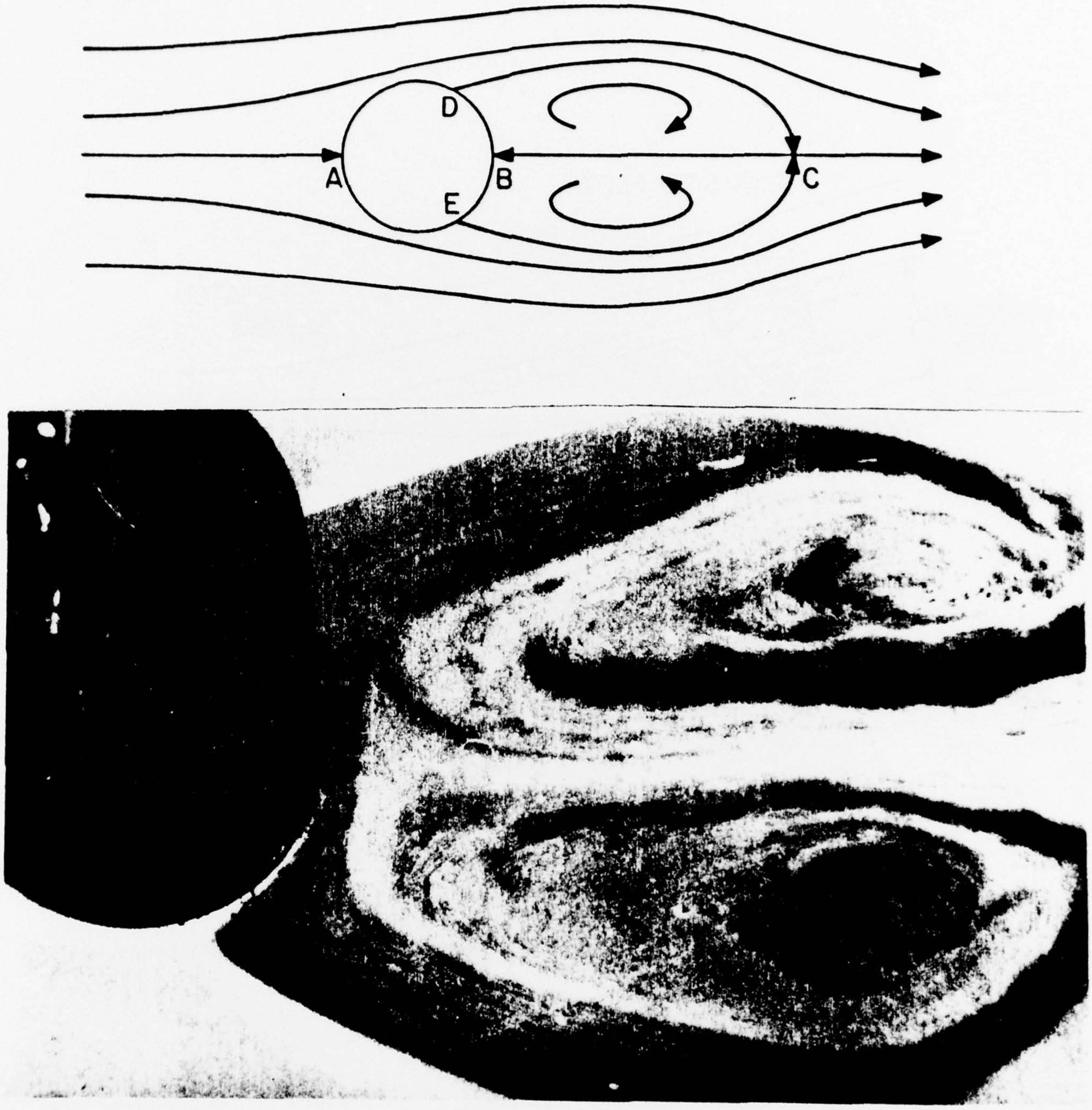

Fig. 8 Schematic sketch and experimental visualization of the streamline pattern of the flow about a circular cylinder ($R \simeq 100$)

Fig. 9 Flow visualization of the immediate neighborhood of separation over a downstream moving wall.

Fig. 10 Flow visualization of the immediate neighborhood of separation over an upstream moving wall.

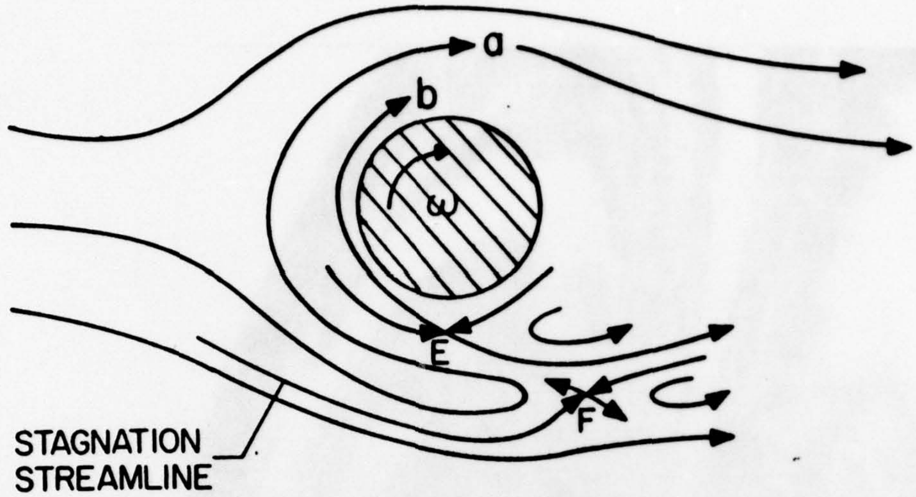

Fig. 11 Schematic sketch of unacceptable streamline patterns for flow over a rotating cylinder.

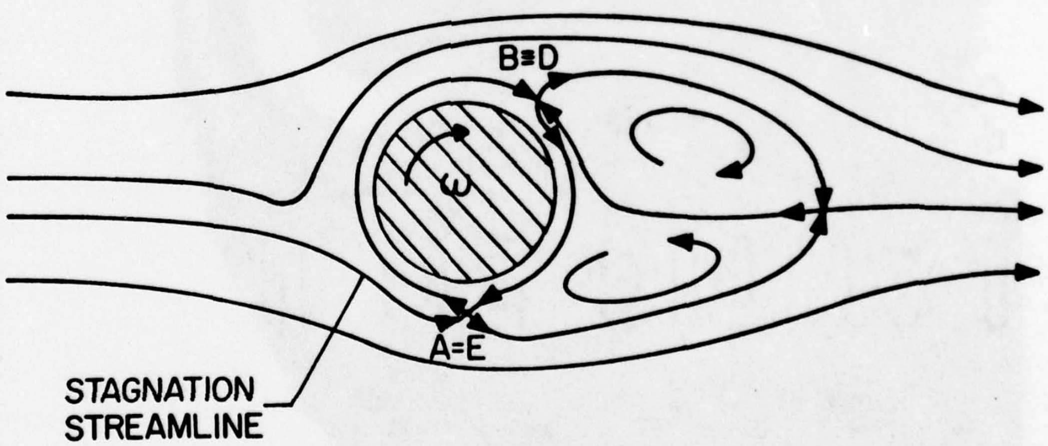

Fig. 12 Schematic sketch of streamline pattern for viscous flow over a rotating cylinder and R = 100 - 500.

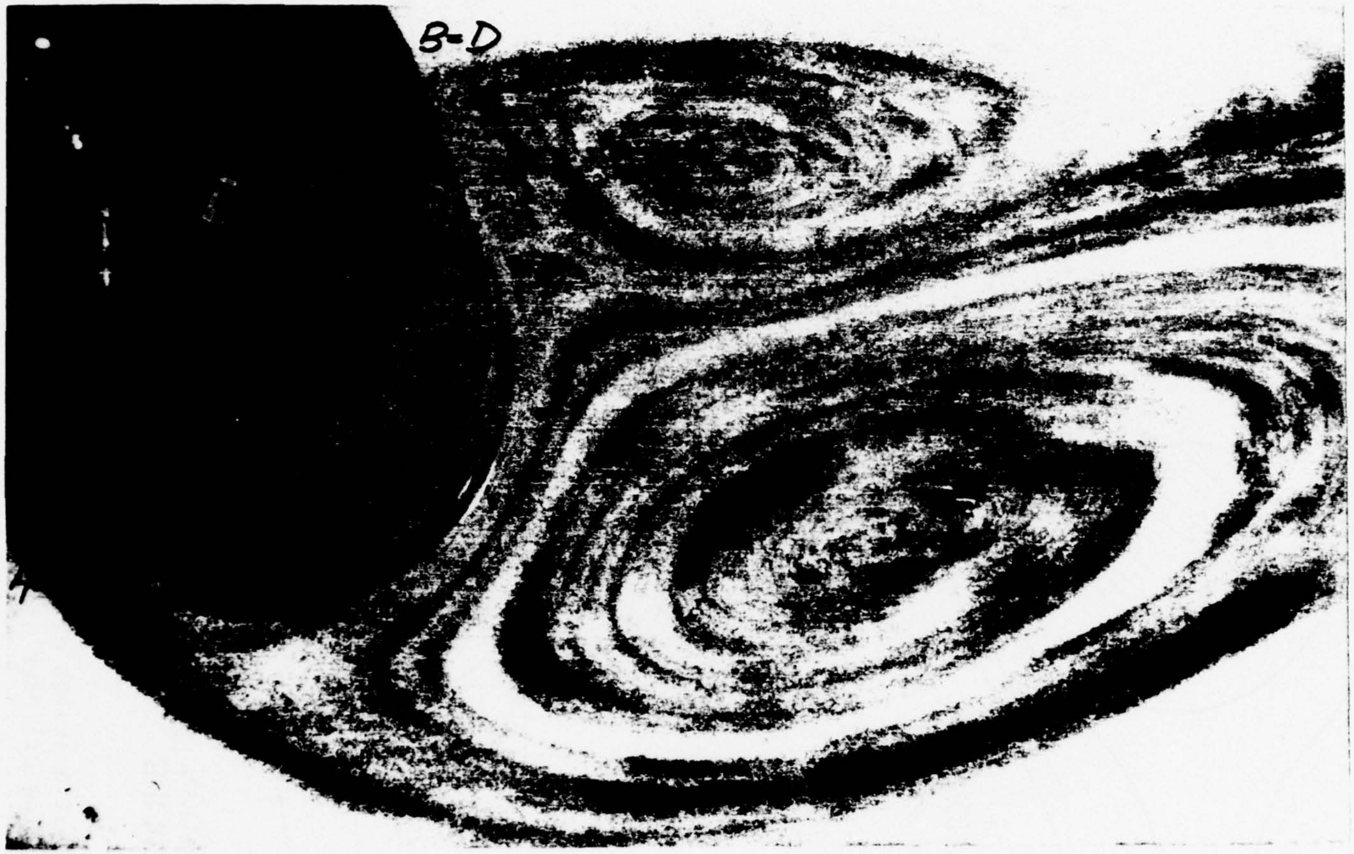

Fig. 13 Flow visualization of streamline pattern for viscous flow over a rotating cylinder. $u_w/U_\infty = 0.2$, $R = 220$.

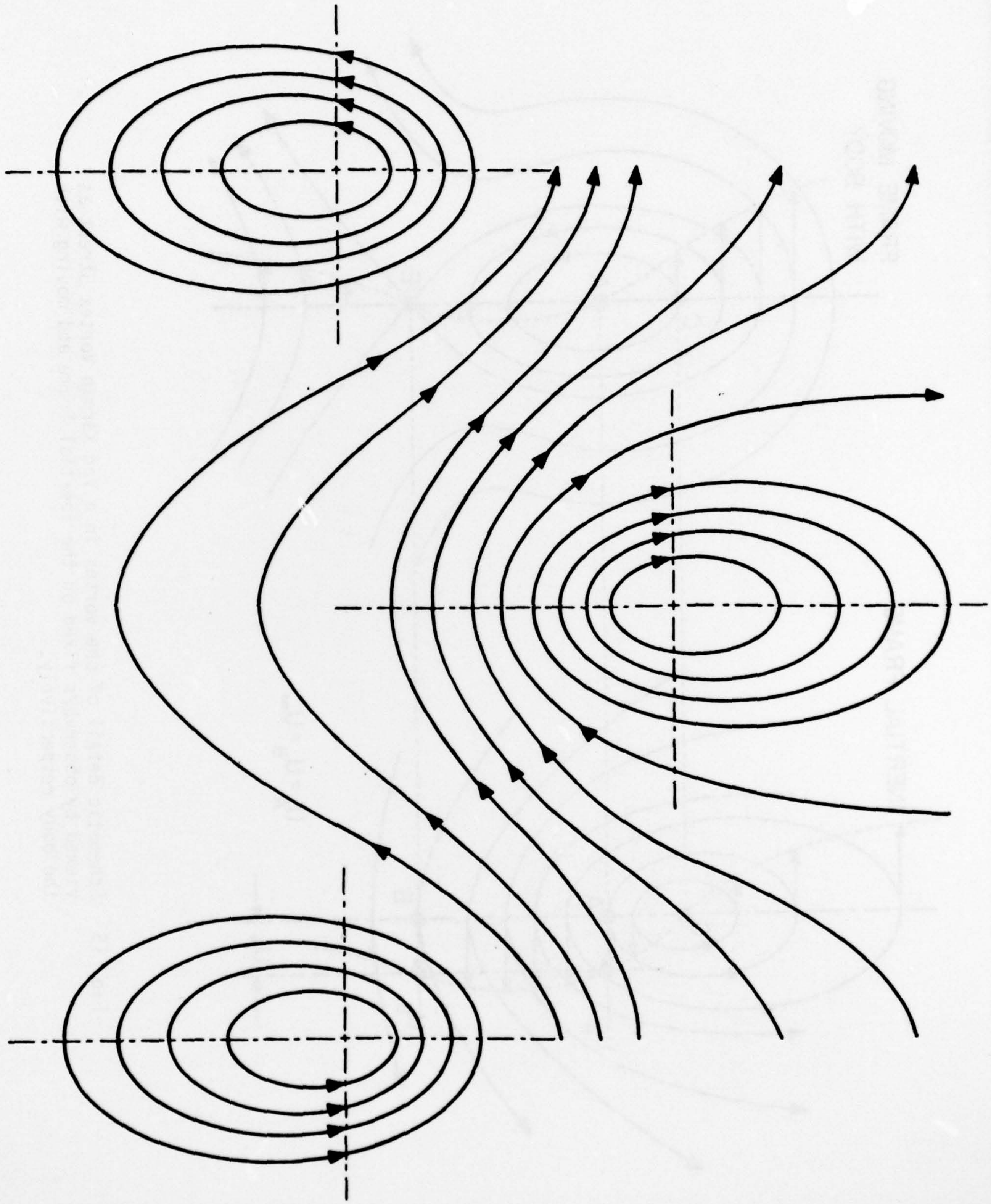

Fig. 14 The Von Kármán Vortex Street for a body moving from left to right as viewed by an observer fixed on the inertial frame.

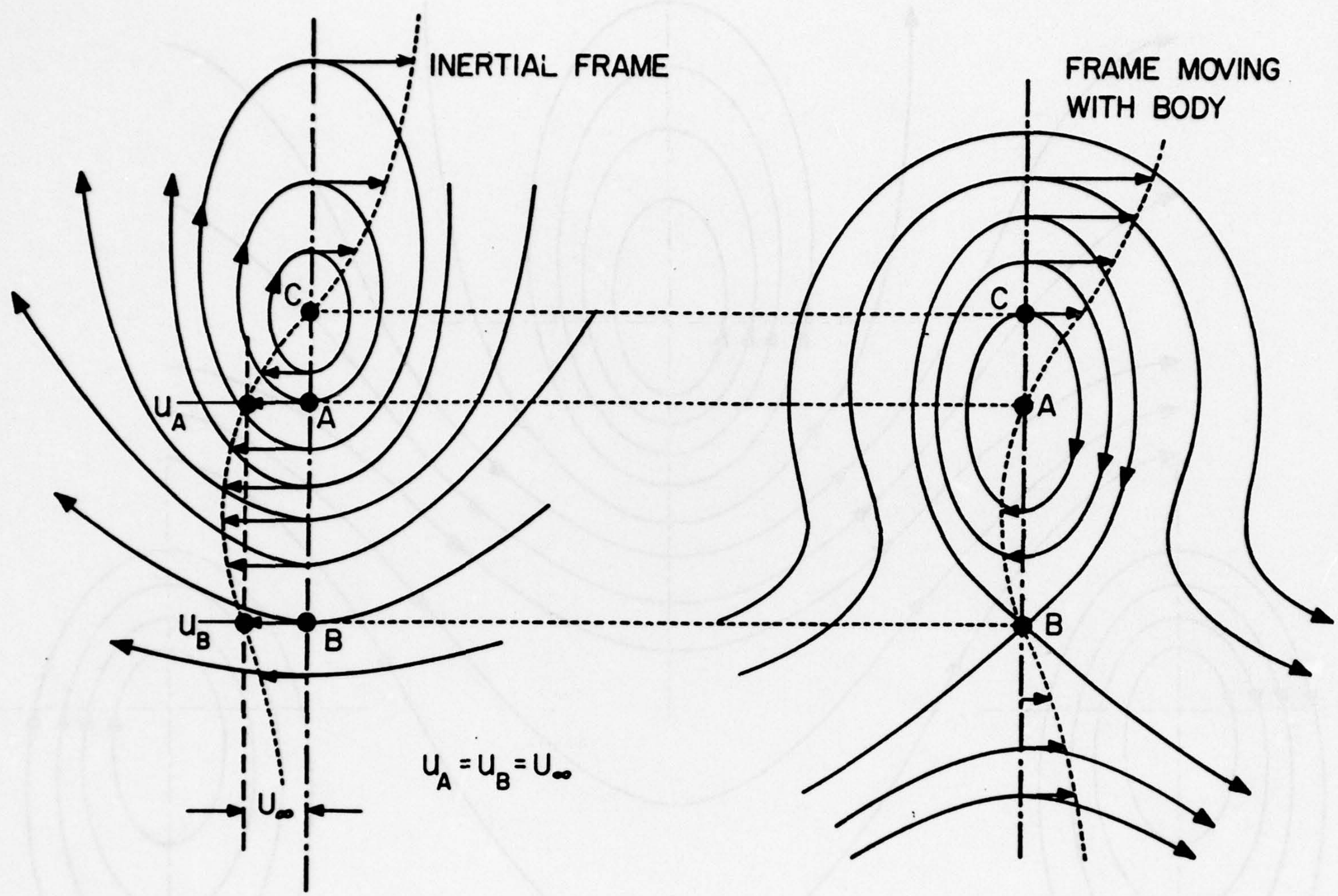

Fig. 15 Schematic detail of one vortex in a Von Kármán Vortex Street as viewed by observers fixed on the inertial frame and moving with the body respectively.

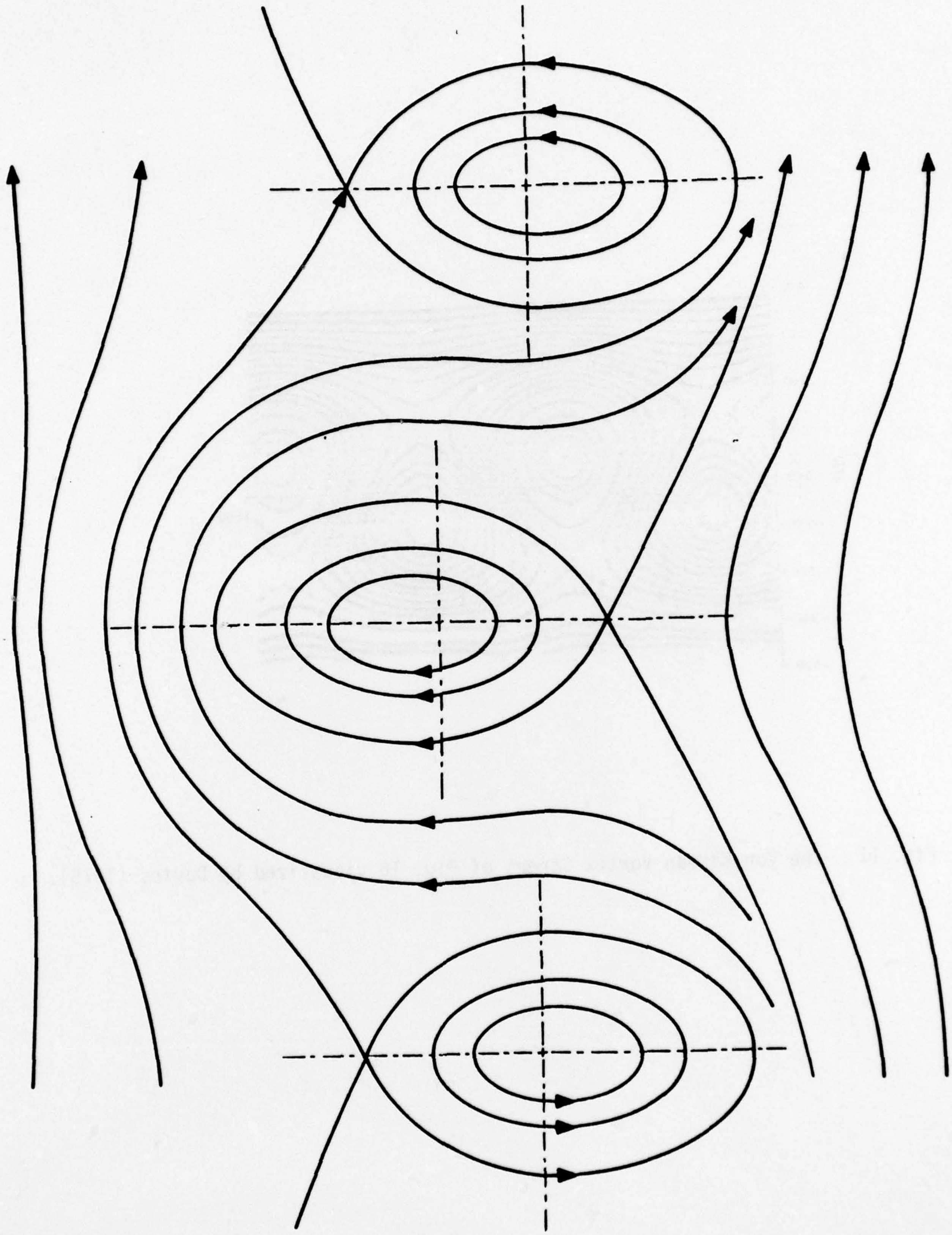

Fig. 16 The Von Kármán Vortex Street as viewed by an observer riding the body.

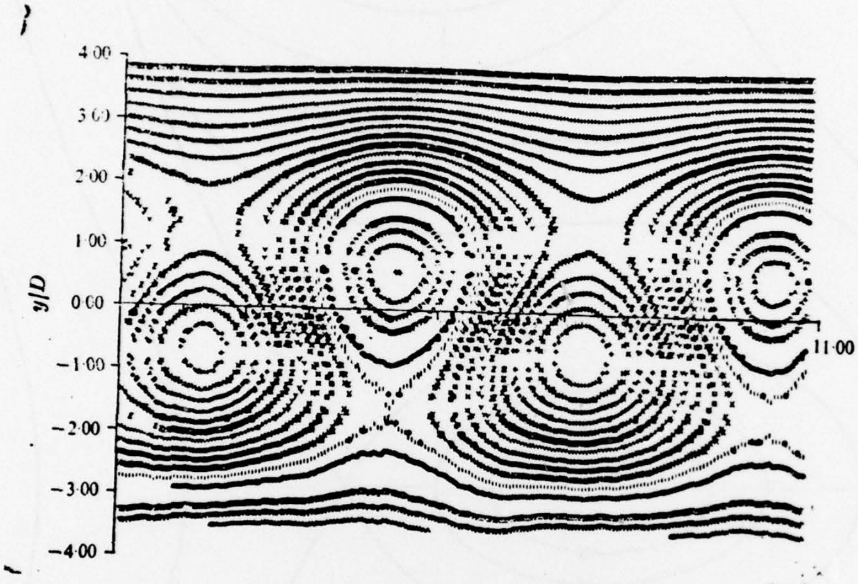

Fig. 17 The Von Kármán Vortex Street of Fig. 16 visualized by Davies (1975).

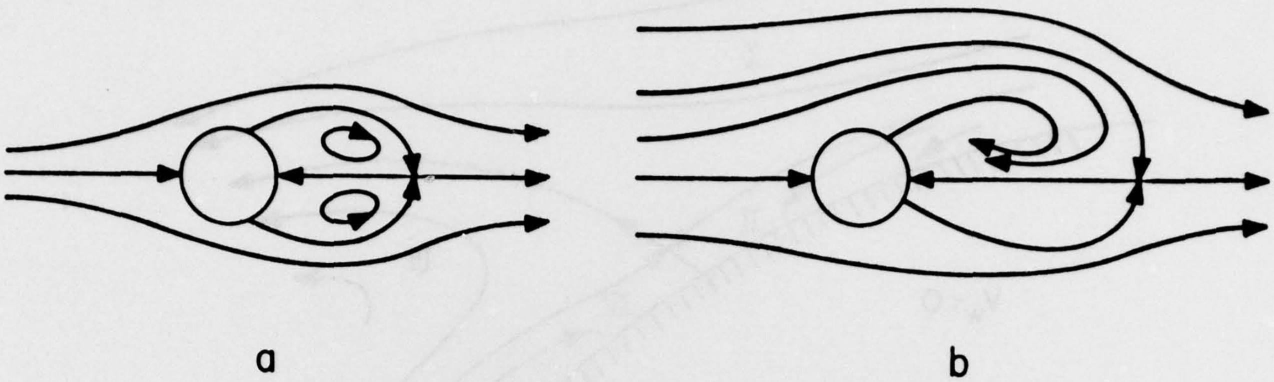

Fig. 18 Streamline pattern of recirculating bubbles. a. Containing a fixed amount of fluid, b. Exchanging mass with free stream.

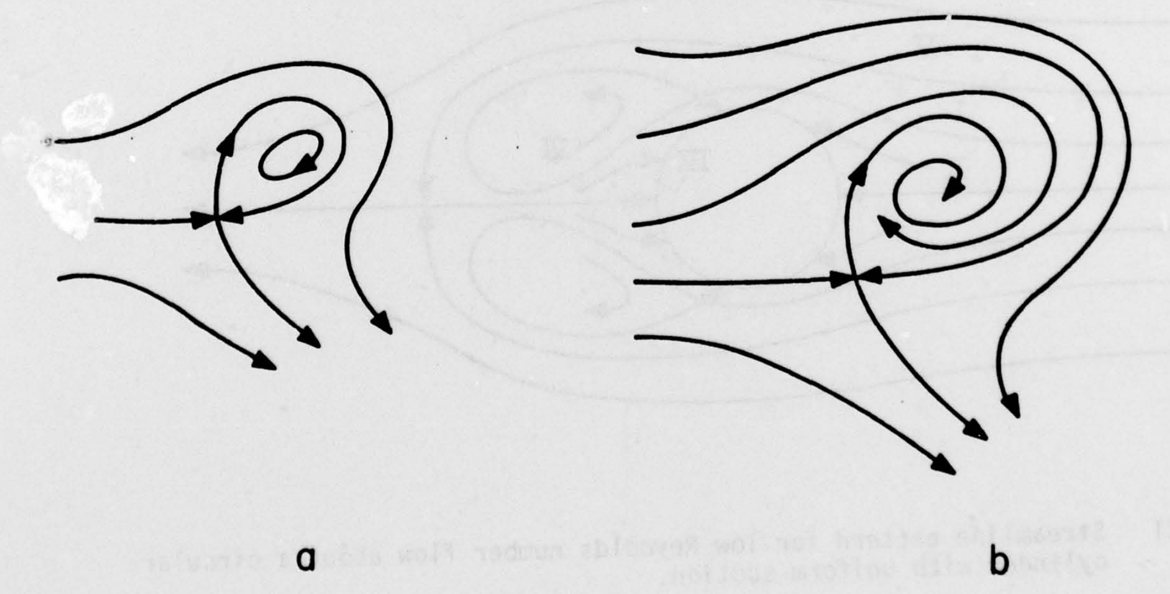

Fig. 19 Streamline pattern of an isolated vortex. a. Containing a fixed amount of Fluid, b. Exchanging mass with free stream.

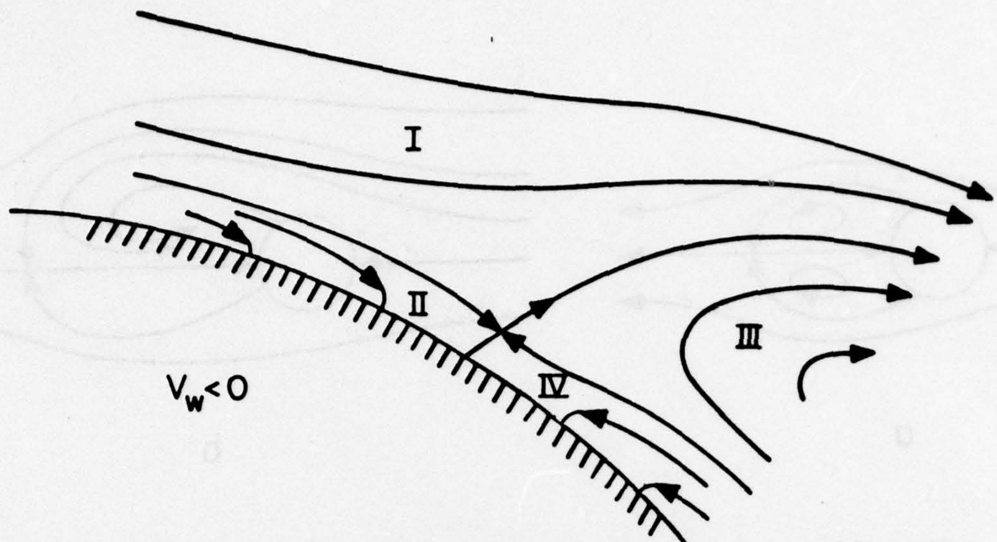

Fig. 20   Streamline pattern for separation over a wall with uniform suction.

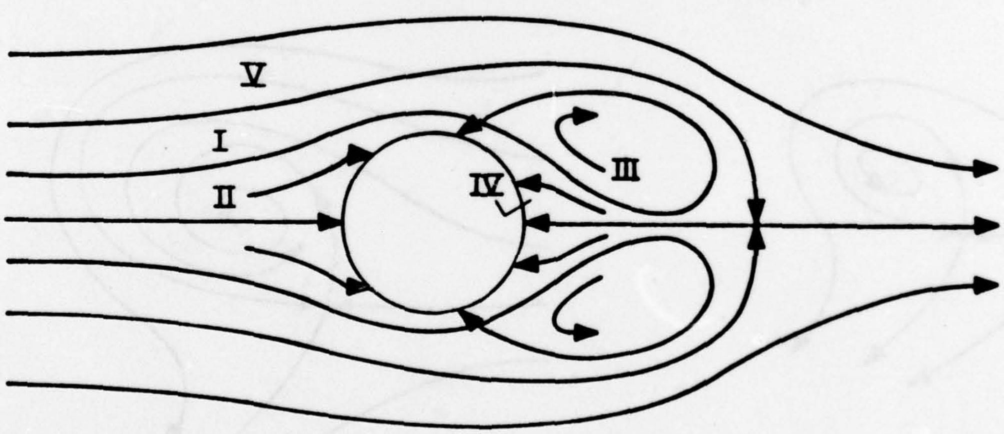

Fig. 21   Streamline pattern for low Reynolds number flow about a circular cylinder with uniform suction.

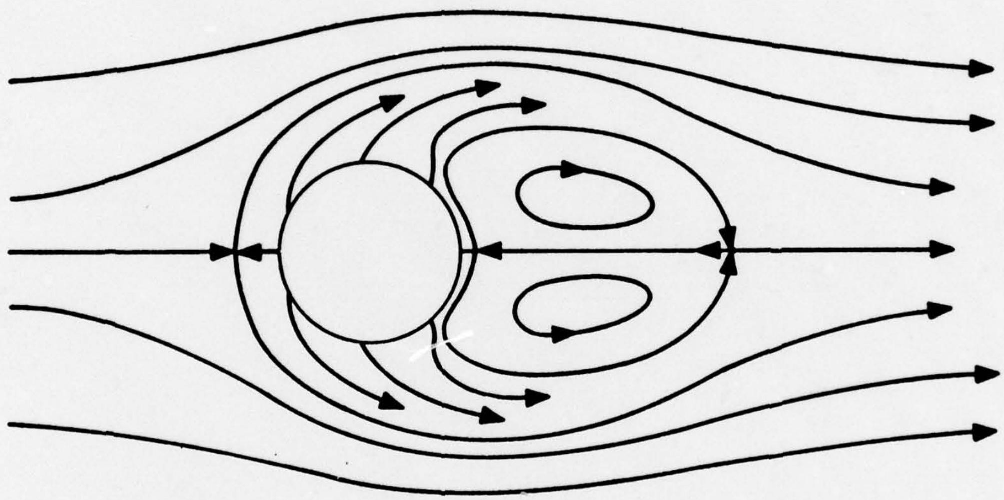

Fig. 22 Streamline pattern for low-Reynolds number flow about a circular cylinder with uniform blowing.

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